Supplement to the paper entitled "On lattice completions and closure operations".

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The present paper contains a new solution of Funayama's second problem, which is of rather constructive character.

Let L be a lattice. Consider a family $\mathfrak{F}$ of subsets of L. When it satisfies the conditions:

(k_1) *if* $X \in \mathfrak{F}$, *then* X *has the* l. u. b.,

(k_2) *if* $X = \{x_\lambda | \lambda \in \Lambda\} \in \mathfrak{F}$ *and* $x_\lambda \leqq y_\lambda \cup z_\lambda$, *then* $\{y_\lambda | \lambda \in \Lambda\} \in \mathfrak{F}$,

then we say that the familly $\mathfrak{F}$ has the property (k).

1. *The familly of all finite (but not empty) subsets of L has the property (k).*

2. *Among all the famillies consisting of subsets of L and having the property (k), there exists a greatest one.*

Proof. Let $\{\mathfrak{F}_\lambda | \lambda \in \Lambda\}$ be the collection of all famillies mentioned above, then $\sum_{\lambda \in \Lambda} \mathfrak{F}_\lambda$ has the property (k).

The gratest familly with the property (k) is denoted by $k(L)$. For $X \subseteqq L$, $k(X)$ denotes the familly consisting of all subsets of X each of which belongs to $k(L)$. By 1, we have

3. *Every finite subset of X belongs to $k(X)$*,

4. Definition.

$$X^k = \{x | x \leqq \sup Y \quad \text{for some} \quad Y \in k(X)\} ,$$

where $X \subseteqq L$.

5. *$\bar{k}$ is a normal closure operation.*

Proof, It is obvious that k is a quasi-closure operation. Then the assertion follows from 3.6 of [1] and 3.

6. *$L \times \bar{k}$ is a sublattice of $L \times i$.*

Proof. It suffices to show that $(A+B)^{\bar{k}} = (A+B)^i$ for any $A, B \in L \times \bar{k}$ by 7.7 of [1]. Suppose that $x \in (A+B)^{ik}$, then there exists $X \in k((A+B)^i)$ such that $x \leqq \sup X$. Let $X = \{x_\lambda | \lambda \in \Lambda\}$, then $x_\lambda \leqq y_\lambda \cup z_\lambda$ where $y_\lambda \in A$ and $z_\lambda \in B$. Denote $Y = \{y_\lambda | \lambda \in \Lambda\}$ and $Z = \{z_\lambda | \lambda \in \Lambda\}$, then $Y \in k(A)$ $Z \in k(B)$ and $x \leqq \sup Y \cup \sup Z$. Hence $x \in (A^k + B^k)^i = (A+B)^i$. Therefore $(A+B)^{ik} \subseteqq (A+B)^i$. Then, by transfinite induction, we have $(A+B)^k \subseteqq (A+B)^{ik} \subseteqq (A+B)^i$, $(A+B)^{k^2} \subseteqq (A+B)^{ik} \subseteqq (A+B)^i, \cdots$, $(A+B)^{k^\alpha} \subseteqq (A+B)^i, \cdots$. Hence $(A+B)^{\bar{k}} \subseteqq (A+B)^i$,

7. *$L \times \bar{k}$ has all principal properties of L.*

Proof. By 7.2 of [1] and 6.

8. *If L is complete, then $L\times\bar{k}$ is isomorphic to L.*

Proof. Since L is complete, $k(L)$ is the familly of all subsets of L. Hence, if $A\in L\times\bar{k}$, then $A=A^k=\{\sup A\}^m$. Then the mapping $A\to\sup A$ establishes the isomorphism between $L\times\bar{k}$ and L.

By 7 and 8, it is shown that $L\times\bar{k}$ *gives a solution of Funayama's second problem.*

Bibliography

[1] Y. SAMPEI, *On lattice completions and closure operations,* this Journal, vol. II (1954), pp. 55–70.

Corrections to "On Lattice Completions and Closure Operations"

By Yoemon SAMPEI

The following corrections refer to the previous paper [1]:

Page 57, line 4, read "$\tau(a)$" instead of "$\tau(\alpha)$".

Page 61, line 5, read "a normal or dual-normal closure operation" instead of "a normal closure operation".

Page 64, line 8 from bottom, read "5.6" instead of "5.7".

Page 64, line 6 from bottom, read "5.7" instead of "5.8".

To the end of section 5, the following few lines are added:

"5.8. *The factor set $\mathfrak{N}(L)$ by isomorphism is a complete lattice.*

Proof. By 5.7 and 3.12".

Page 65, lines 9, 10, 12, and page 66, lines 3 and 4, read "MacNeille's" instead of "MacNeil's".

Page 66, line 9 from bottom, read "or of" instead of "and of".

Reference

[1] Y. SAMPEI. *On lattice completions and closure operations,* this Journal, vol. II (1954), pp. 55–70.

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